# B.Sc. (Honours) Examination, 2019
## Semester-III (CBCS)
## Statistics
## Course : CC-5
## (Sampling Distribution)

**Time : 3 Hours** **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a. State and prove Lindeberg-Levy Central Limit Theorem (CLT). Hence or otherwise prove DeMoivre-Laplace theorem.
   b. Check whether Weak Law of Large Numbers (WLLN) holds for the following sequence of random variables: 6+4=10
   $$P(X_n = n) = P(X_n = -n) = \frac{1}{2}n^{-\frac{1}{2}}, P(X_n = 0) = 1 - n^{-\frac{1}{2}}$$
2. a. Show that $X_{(k)}$in a random sample of size $n$ from a $R(0,1)$distribution has a beta distribution with parameters $(k, n-k+1)$.
   b. Show that the pdf of the sample range from an $R(0,1)$ distribution is given by $n(n-1)r^{n-2}(1-r), 0 \le r \le 1, r(range) = x_{(n)} - x_{(1)}$. 3+7=10
3. a. Show that if $X$ and $Y$are normal variables with zero means, unit variances and correlation coefficient $\rho$, then
   $$E[\max(X,Y)] = \sqrt{(\frac{1-\rho}{\pi})}$$
   b. Show that the sample variance $S^2$obeys the equation
   $$(n-1)S^2 = \sum_{i=2}^{n} \frac{i(X_i - \bar{X}_i)}{i-1}$$ 5+5=10
   where $\bar{X}_i = \frac{X_1+X_2+\cdots+X_i}{i}$
4. Let $X_1, X_2, \ldots X_n$be $n$ random variables distributed as $N(0,1)$independently. Find the distribution of $\chi^2 = X_1^2 + X_2^2 + \cdots + X_n^2$. Also find $E(\chi^2)$ and $V(\chi^2)$. Also discuss the case when $n \to \infty$. 10
5. a. State and prove Markov's inequality. Hence or otherwise prove Chebyshev's inequality.
   b. Suppose $X$ has mean $\mu$ and non-zero variance $\sigma^2$. Show that
   $$P(X \le x) = \frac{\sigma^2}{\sigma^2 + (x-\mu)^2} \text{ for } x \le \mu \text{ .}$$ 6+4=10
6. a. Let $X_1$and $X_2$be independently binomially distributed random variables, with parameters $(n_1, \frac{1}{2})$ and $\left(n_2, \frac{1}{2}\right)$, respectively. Show that $X_1 - X_2 + n_2$ has the binomial distribution with parameters $(n_1 + n_2, \frac{1}{2})$.

P.T.O.

b. Let $X$ and $Y$ be independently distributed, each in the form $N(0,1)$. Show that $Z = X/Y$ has the Cauchy distribution with pdf

$$f(z) = \frac{1}{\pi[1+z^2]}$$

What would be the distributions of $W_1 = X/|Y|$ and $W_2 = X/|X|$? 4+6=10

7. a. Write down the test procedure to perform a large sample test for comparing two independent binomial proportions.

b. Hence or otherwise find a $100(1-\alpha)\%$ confidence interval for the difference of proportions. Find the expected length of the interval.

6+4=10